

Lastly, on the question whether the peculiarities of structure recognized respectively in *N. Pompilius* and *N. umbilicatus* are sufficient to establish a difference of species, or are attributable merely to variety, the author observes, that any tendency in a being to revert to an original type, when such has been determined, betrays variety; but this tendency is never manifested in the *Nautili* under consideration by the occasional occurrence of specimens presenting characters which place them intermediately between *N. Pompilius* and *N. umbilicatus*. Having visited the Fijii Islands since he formerly wrote on *N. Pompilius*, he finds that the umbilicated *Nautili* are not known to the natives, although *N. Pompilius* is very plentiful; but at Fatuna or Wallis's Island, where both are found, the people recognize the difference between them depending on the presence or absence of umbilical pits. On this the author remarks, that although particular localities, with all attending circumstances, may favour the production of varieties, yet the permanence of the distinctive characters of these *Nautili* without symptom of amalgamation, and the discovery of a female specimen of *N. umbilicatus*, are strong arguments in support of the view that they are distinct species, though very closely allied.

Further descriptive details are given in the explanation of the figures which accompany the memoir.

III. "On a Class of Differential Equations, including those which occur in Dynamical Problems."—Part II. By W. F. DONKIN, M.A., F.R.S., F.R.A.S., Savilian Professor of Astronomy in the University of Oxford. Received February 17, 1855.

This is the second and concluding part of a paper of which the first part was printed in the Philosophical Transactions for 1854. In the fourth section (the first of this part) some of the most important results of the former part are recapitulated.

In the fifth section the theory of the Variation of Elements is considered under that aspect which belongs to it in connexion with the general methods of this paper; and the facility of its application is shown in two instances: (1) the expressions for the variations of

the elliptic elements of a disturbed planet's orbit are deduced from the results of Art. 30 (Part I.), on undisturbed elliptic motion ; (2) the problem of determining the motion of a free simple pendulum (omitting the effect of the earth's rotation) is treated by considering the orbit of the projection of the bob upon a horizontal plane as a disturbed ellipse. The differential equations which define the variations of the elements of the ellipse are given in a rigorous form, and integrated approximately so as to give the motion of the apsides of the mean ellipse in any case where the pendulum never deviates much from the vertical, and the motion is not very nearly circular. The result agrees with the conclusions of the Astronomer Royal (Proceedings of the Royal Astronomical Society, vol. xi. p. 160).

In the fifth section the transformation of the differential equations by the substitution of new variables is considered, and particularly that kind of transformation, called by the author a *normal transformation*, which leads to a new system of equations, not merely possessing the same general form as the old, but distinguished also by other common properties. A definition is given of those transformations which may be properly called, from analogy, *transformations of coordinates*, and it is shown that all transformations of coordinates are *normal*. General formulæ are given for transforming the equations of any dynamical problem from *fixed* or *moving* systems of axes of coordinates ; and an illustration is drawn from the case of the motion of a planet referred to axes in the varying plane of its own orbit.

In the seventh and last section the principles of transformation developed in the preceding section are applied in a more general manner to the differential equations of the planetary theory ; and it is shown that when the motions of a planetary system are referred to a system of rectangular axes having their origin in the sun, and otherwise moving in any arbitrary manner, the variations of the elements will still be determined by the same formulæ as if the axes were fixed, provided there be added to the disturbing function R , for each planet, the expression

$$\sqrt{\mu a(1-e^2)} \cdot (\omega_0 \sin \nu \sin \iota - \omega_1 \cos \nu \sin \iota + \omega_2 \cos \iota),$$

in which ι is the inclination of the orbit to the (moving) plane of xy , the longitude of the node reckoned from the axis of x , and $\omega_0, \omega_1, \omega_2$;

the angular velocities of the moving system of axes about the three axes themselves. In this expression $\omega_0, \omega_1, \omega_2$ may be any arbitrary functions either of the time or of the elements; but in any case these functions are to be exempted from differentiation with respect to the elements in taking the partial differential coefficients of the disturbing function.

This result is illustrated by referring the motion of a system consisting of *two* planets to axes so chosen that the plane of *xy* shall always coincide with the *principal plane* of the system, and the axis of *x*, from which all longitudes are reckoned, shall always coincide with the line of nodes; there are thus obtained twelve rigorous simultaneous differential equations, of which nine form a system apart, containing only the major axes, excentricities, epochs, longitudes of perihelia, and mutual inclination of the two orbits, and afford an example of the so-called "elimination of the nodes;" whilst the remaining three (which contain also these nine elements, but not their differential coefficients) determine the motion of the principal plane and of the line of nodes, relatively to fixed space. The mutual inclination of the two orbits being supposed known, their several inclinations to the principal plane are given by simple relations; and the positions of the planets in their orbits (their longitudes reckoned from the line of nodes) being supposed known, their motions relatively to fixed space would thus be completely determined.

IV. Extract of a Letter, dated January 6, 1855, from J. MITCHELL, Esq., Quartermaster of Artillery, Bangalore, "On the Influence of Local Altitude on the Burning of the Fuses of Shells."

"In the early part of the year 1848, at the annual practice of the Artillery in this garrison, it was observed that the fuses burned too long a time. The regular burning of fuses being a matter of much importance, the circumstance was duly reported to Artillery Head Quarters, and a portion of each kind was directed to be sent to St. Thomas's Mount (eight miles from Madras and on the same